The future of Oil

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Abstract

In this paper, we consider the future of oil. Starting with older predication methods we eventually developed the intuition we used until we came up with a few systems of differential equations ourselves. Some of these differential equations could be reduced to functions relating quantity of oil with respect to time, while others couldn't be reduced to such functions.

1 Introduction

For decades there has been fear that our highest used transportation fuel, oil, would run out. Much of the attention to the exhaustion of oil was brought about because of M. King Hubbert, who in 1956 used a bell curve to predict oil peak production as well as it's exhaustion in the United States [6]. The fact that his predicted year of peak production of oil, 1970, was correct, brought much attention to modeling oil production. The most recent publications have either found, or stated that we are about to hit, or have passed the world oil production peak. One such paper by Maggio and Cacciola [7] found that the world oil peak is within the next 2 years or so. Maggio and Cacciola used a modified version of Hubbert's model to get their estimate. Likewise, Castro, Miguel and Mediavilla [4] who posted their findings in 2009, found that the peak would occur around 2011. These three worked on this problem from a more economical view, wherein they related oil production to GDP. They also try to relate the differences between conventional and non-conventional oil production. For future reference cumulative production data for 1978 was taken from Gallagher [5] added to the annual production data of 1979 from the BP report [2], resulted in the cumulative of 1979. Following this, the annual production and reserve data was found from BP [2], and cumulative for a given year was found by: $Q_i = Q_{i-1} + P_i$.

2 Reserve-to-production(R/P) Model

The paper starts with an introduction to the first and easiest form of estimating oil production. This model is the reserve-to-production (R/P) model. The amount of years remaining until reserve is depleted is calculated by an estimated reserve(R) divided by current production(P). Brandt (2010) said that one of the earliest users of this model is Day which occured in 1909 [3]. The main problem that this model introduces is that the actual calculated amount for reserves changes as we find more oil fields. Furthermore the quantity of reserves does not take into account the fact we can extract more oil through technological advances. Furthermore, production is considered constant in this model, but we have seen that production changes overtime, sometimes increasing and sometimes decreasing depending on the given year. For example worldwide oil proved resources for 1980 is believed to be 683.4 billion barrels, and 1981 to be 696.5 billion barrels, while worldwide production for 1980 was 22.97 billion barrels. This means that on top of of covering the production of 22.97 billion barrels, the reserves also increased by 13.2 billion barrels. This leads to an increase of reserves for that year of 36.17. Ignoring this discrepancy, using the R/P method for 1980 there should be 29.75 years remaining, or in other words oil would be exhausted in the later half of the year 2009. While using this method with the values of reserve and production for 2014, 1700.1 billion barrels and 32.37 respectively, gives 52.52 years remaining of oi, or exhaustion in the later half of the year 2066. This means that using this model we have extended our date of exhaustion by 57 years, even though we are producing more.

3 Hubbert Logistic Model

Before we introduce the Hubbert logistic model, we will first show a simpler model, the exponential model. The exponential model, is a model where as time grows, the production increases by a rate proportional to the amount already pumped. Or in other words:

$$Q'(t) = rQ(t) \tag{1}$$

where r is a rate, $Q(t)$ is the total quantity pumped at time t. By integrating this we are lead to the following estimation for cumulative quantity of oil.

$$Q(t) = Q_0 e^{rt} \tag{2}$$

The issues with this model are many, but one of the largest, is that as time increases cumulative quantity increases. This would be a nice result to have, as it means that production of oil would be boundless, we could keep producing forever. This brings us to the next model and the main topic of this section, Hubbert's logistic model.

The Hubbert model, is a logistic model. Hubbert first used this to model in 1956 to predict the peak of oil. He predicted that US oil production would peak somewhere between 1965 and 1970. American oil production peaked in 1970. This model starts with the following equation, where $Q(t)$ is the cumulative production at time t, r is a proportionality constant with units of $\frac{years}{barrels}$, and Q_∞ is the predicted amount of oil that is available to be used, plus that which has already been used:

$$Q'(t) = rQ(t)(Q_\infty - Q(t)) = P(t) \tag{3}$$

$$Q(t) = \frac{Q_\infty c e^{rtQ_\infty}}{1 + c e^{rtQ_\infty}} \tag{4}$$

Eq. (8) is sometimes written as Eq. (9), if we let $a = \frac{1}{c}$, and divide both top and bottom by ce^{rtQ_∞} :

$$Q(t) = \frac{Q_\infty}{1 + a e^{-rtQ_\infty}} \tag{5}$$

This results in a symmetric production curve, or P(t) curve. The actual production curve for the United States does not appear to be symmetric, as the decrease on the right is slower than the increase on the left of the peak production, as seen by P. Berg and S. Korte [1]. Some other limitations on the Hubbert model, as proposed by Gallagher [5] is that even if we assume it is correct for worldwide, or some countries, it does not properly model specific examples. Gallagher gives Illinois as one such example, as Illinois has three phases where it appears to be Hubbert, but overall it looks completely not Hubbert. Furthermore, the goal of this paper is to consider the Worldwide oil production, not just the American. We cannot tell if the Worldwide oil production would be symmetric, so the following equations consider it need not be.

4 Introduction to Reserves

The Reserves model could be considered a transition to a system of differential equations that considered both reserves and cumulative production. In this model, the change of reserves is just considered to be the negative value of production. The reason for this is it considers only considers reserves to be decreasing when oil is produced.

$$Q'(t) = rQ(t)(Q_\infty - Q(t)) = P(t) \tag{6}$$

$$R'(t) = -P(t) \tag{7}$$

Eq.(10) and Eq. (11) lead to Eq. (12) through same process as for the Hubbert model.

$$Q(t) = \frac{Q_\infty}{1 + ae^{-rtQ_\infty}} \tag{8}$$

Which will result to the same cumulative production model as the Hubbert model, which will cause the same errors and issues. Furthermore, in this model reserves is strictly decreasing, but given the data we have the reserves for the year 1980 was 683.4 billion barrels, but had increased to 1700.1 billion barrels in 2014. This is clearly not decreasing, but instead is increasing. So this model for the growth of reserves is clearly wrong as well.

5 System of differential equations: Reserves and Cumulative production

This system of differential equation is based around the fact that cumulative production grows by how much cumulative production there is, Q, and how much oil there is remaining or in other words the reserves, R. This would be close to the Hubbert Model in theory, as $(Q_\infty - Q(t))$ is the total amount of oil remaining in the ground, which is approximately $R(t)$ The growth of reserves, $R'(t)$ is decreased by P(t), and a factor that is similar to the logistic model. In this case we have Eq. (13) is cumulative production and Eq. (14) is the reserves.

$$\frac{dQ}{dt} = rQR = P \tag{9}$$

$$\frac{dR}{dt} = -P + \alpha(1 - \beta R)R \tag{10}$$

In these two equations, r, α, and β are all constants. Whereas R means the reserves at time t, Q is cumulative production at time t, and $\frac{dQ}{dt}$ and $\frac{dR}{dt}$ are the rate of change of cumulative product and reserve respectively. From here, finding R(Q) is not extremely difficult here, but on the other hand R(t) and Q(t) are difficult to get in all but some very specific cases. The following equation is found by replacing P in Eq. (14) and distributing α.

$$R'(t) = -rQR + \alpha R - \alpha\beta R^2 \tag{11}$$

Using the chain rule on $R'(t)$ and the reciprocal of $Q'(t)$ results in:

$$\frac{dR(t)}{dQ(t)} = -1 + \frac{\alpha}{rQ(t)} - \frac{\alpha\beta R(t)}{rQ(t)} \tag{12}$$

Bringing the $\frac{\alpha\beta R}{rQ}$ term over to the other side we get:

$$\frac{dR(t)}{dQ(t)} + \frac{\alpha\beta R(t)}{rQ(t)} = -1 + \frac{\alpha}{rQ(t)} \tag{13}$$

Multiplying the whole equation by $Q(t)^{\frac{\alpha\beta}{r}}$ we get:

$$\frac{dR(t)}{dQ(t)}Q(t)^{\frac{\alpha\beta}{r}} + \frac{\alpha\beta R(t)Q(t)^{\frac{\alpha\beta}{r}+1}}{r} = -Q(t)^{\frac{\alpha\beta}{r}} + \frac{\alpha Q(t)^{\frac{\alpha\beta}{r}}}{r)} \tag{14}$$

Solving this resolves to:

$$R(Q) = \frac{1}{\beta} - \frac{Q(t)}{\frac{\alpha\beta}{r}+1} + \frac{c}{Q(t)^{\frac{\alpha\beta}{r}}} \tag{15}$$

To find the best constants for this model we use the sum of squared errors method, but having substituted the following constants, $\frac{1}{\beta} = b$ and $\frac{\alpha\beta}{r} = a$ this reduces our equation to:

$$R(Q) = b - \frac{Q(t)}{a+1} + \frac{c}{Q(t)^a} \tag{16}$$

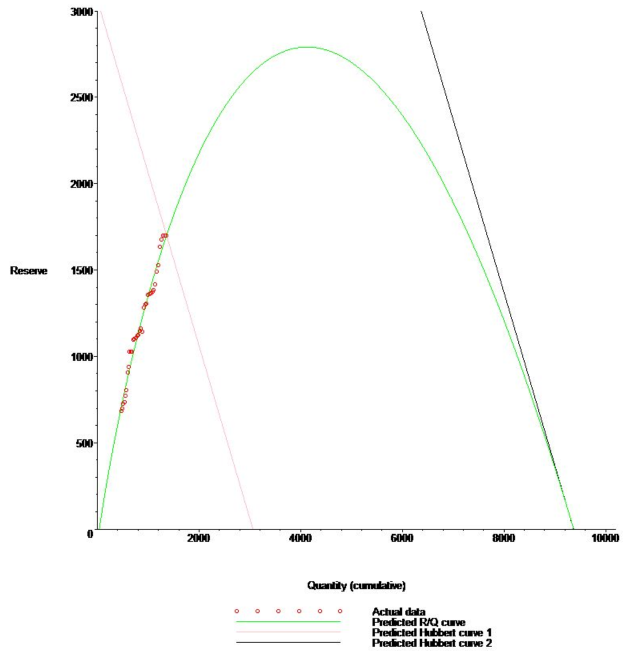

Figure 1: Our first model, in the R over Q plane

After using the sum of squared error method, we find that the following are constants that produce the lowest sum of square error: α=0.003096606, β= -0.013640377, c= -0.040497328 and r= 0.000029333. Using these values, we found the following graph.

The graph (1) is a graph where the left axis is Reserves and the bottom axis is cumulative quantity produced. Notice that the determined value for Q_∞, or the predicted amount of oil that will be found and used is approximately 9500 billion barrels. This amount is five or six times our current amount, but this graph has no scale of time, so it could quickly grow to become that point in twenty years, or a hundred. To compare our model to the Hubbert model, the pink is a graph in the RQ plane if we took Q_∞ to be the current amount reserves added to the current amount of cumulative production, in other words no more reserves found. The black is a Hubbert model in the RQ plane if we took Q_∞ to be predicted Q_∞ for our model. As can be easily seen, Hubbert model does not corrrectly predict the relation between reserves and cumulative production. It is not even close, as the two are heading in completely different directions. Even if it properly models oil

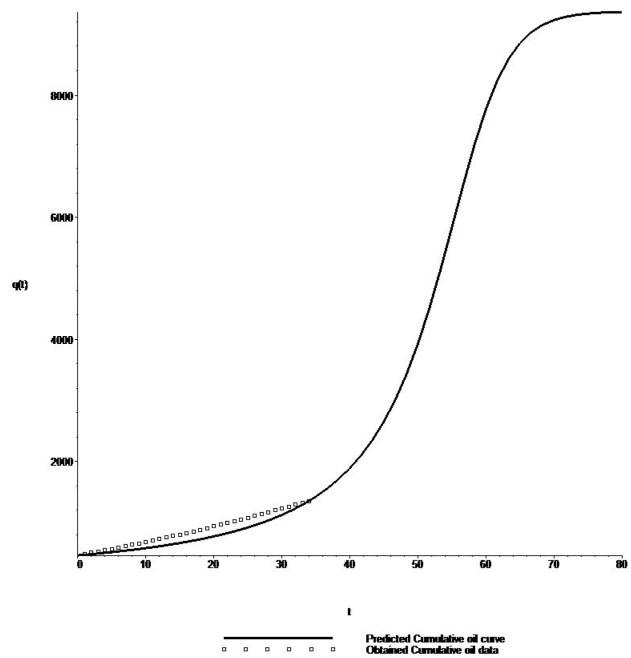

Figure 2: Our first model's curve of Q 0 in time is 1980

production in the RQ plane, it would only do so after we have completely stopped finding more oil reserves. This means that the Hubbert model would not even correctly predict more than half of the data given, and would only correctly predict the decline, if anything.

Below (2) is another graph, that was found using the DEplot function in maple on our Differential equation of $Q'(t)$ after plugging in our previously stated constants for α, β, c, and r. The line in this space follows the data quite well, though, if this model were correct, it would predict a sharp increase of oil production in the next few years.

6 System of differential equations: Reserves over production

Following the above system of differential equations, we decided to modify the relationship involving $\frac{dR}{dt}$. So we modified it to have depend on reserves over production instead of just purely on reserves. This change in the relationship was caused mainly because we thought that reserves should grow should be based upon how much we are producing as well. While production is low, it does not change much from our earlier model, but when production gets larger, it starts to make reserves change more drastically. Doing this change, actually made the resulting system of differential equations far easier to solve. Below, the modified system of differential equations are shown as well as the work they produced.

$$\frac{dQ}{dt} = rQR = P \tag{17}$$

$$\frac{dR}{dt} = -P + \alpha(1 - \beta\frac{R}{P})R \tag{18}$$

Using these two equations to find $\frac{dR}{dQ}$ results in:

$$\frac{dR}{dQ} = -1 + \frac{\alpha}{rQ} - \frac{\alpha\beta}{r^2Q^2} \tag{19}$$

After this let $a = \frac{\alpha}{r}$ and $b = \frac{\alpha\beta}{r^2}$ doing so creates the following equation:

$$\frac{dR}{dQ} = -1 + \frac{a}{Q} - \frac{b}{Q^2} \tag{20}$$

Bringing the dQ to the other side and integrating both sides: Integrating reduces the equation to the following:

$$R(Q) = -Q(t) + a\ln(Q(t)) + \frac{b}{Q(t)} + c \tag{21}$$

Following this, using the equation for sum of squared errors, we found the optimal constants that satisfy the sum of squared errors, $SSE = \sum(R(Q_i) - R_i)^2$. Consider the following equations:

$$\frac{dSSE}{da} = 2 * \sum(-Q_i + a\ln(Q_i) + \frac{b}{Q_i} + c - R_i)ln(Q_i) \tag{22}$$

$$\frac{dSSE}{db} = 2 * \sum(-Q_i + a\ln(Q_i) + \frac{b}{Q_i} + c - R_i)\frac{1}{Q_i} \tag{23}$$

$$\frac{dSSE}{dc} = 2 * \sum(-Q_i + a\ln(Q_i) + \frac{b}{Q_i} + c - R_i) \tag{24}$$

setting these equal to zero, and isolating the values solely dealing with R_i and Q_i on one side, while we have the remaining on the other leads to:

$$a\sum(\ln(Q_i)^2)+b\sum\frac{\ln(Q_i)}{Q_i}+c\sum\ln(Q_i)=\sum(\ln(Q_i)R_i))+\sum(\ln(Q_i)(Q_i)) \tag{25}$$

$$a\sum\frac{\ln(Q_i)}{Q_i}+b\sum\frac{1}{Q_i^2}+c\sum\frac{1}{Q_i}=\sum\frac{R_i}{Q_i}+n \tag{26}$$

$$a\sum\ln(Q_i)+b\sum\frac{1}{Q_i}+cn=\sum R_i+\sum Q_i \tag{27}$$

These three equations together form a system of equations with three equations and three unknowns, a, b, c.

$$\begin{pmatrix} \sum(\ln(Q_i)^2) & \sum\frac{\ln(Q_i)}{Q_i} & \sum\ln(Q_i) \\ \sum\frac{\ln(Q_i)}{Q_i} & \sum\frac{1}{Q_i^2} & \sum\frac{1}{Q_i} \\ \sum\ln(Q_i) & \sum\frac{1}{Q_i} & n \end{pmatrix}$$

$\times$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$=$

$$\begin{pmatrix} \sum(\ln(Q_i)R_i))+\sum(\ln(Q_i)(Q_i)) \\ \sum\frac{R_i}{Q_i}+n \\ \sum R_i+\sum Q_i \end{pmatrix}$$

Substituing the values for Q_i and R_i for each of the equations below and then simplifying this matrix reduces it to:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2945.39 \\ 0 & 1 & 0 & 919550.46 \\ 0 & 0 & 1 & -18898.47 \end{array}\right] \tag{28}$$

Setting the values found in the matrix above as our a, b, c in the following equation from above, $R(Q)=-Q(t)+a\ln(Q(t))+\frac{b}{Q(t)}+c$, we are given the graph below, with y-axis being reserves (R) and x-axis being cumulative quantity produced (Q).

The graph appears to hold a close approximation to our data points. This implies that this graph would likely hold true for values lower than those

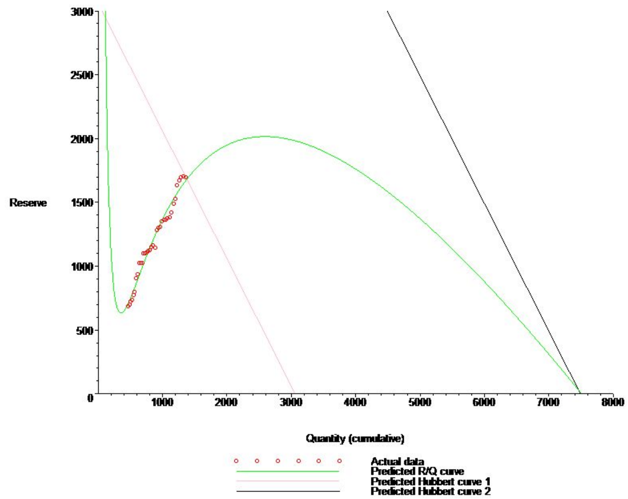

Figure 3: Our second model's curve of Reserves and Cumulative Quantity

within the 1980 to 2014 dataset. This model estimates the Q_∞ to be roughly 7550 billion barrels of oil. This is roughly four and a half times our currently cumulative oil produced. In the abovegraphical representation for this model, we impose the Hubbert model in the above graphical representation as the two linear lines. The pink line being if we currently have an accurate value for the amount of total reserves remaining. While the black line is a line used if the total amount of oil pumped was our estimated amount, and that Hubbert's model was correct. There is still no changes from above, the Hubbert model definitely does not predict the proper reserve to cumulative production graph.

Substituing the obtained equation, along with its constants, R(Q) into our original differential equation for the change in cumulative production returns:

$$Q'(t) = rQ(t)(-Q(t) + a\ln(Q(t)) + \frac{b}{Q(t)} + c) \tag{29}$$

Bringing back in our substitution for a and b, then we get the following

$$Q'(t) = -rQ(t)^2 + \alpha Q(t)\ln(Q(t)) + \frac{\alpha\beta}{rQ(t)} + rcQ(t) \tag{30}$$

Simplifying the equation, and then taking the integrating returns:

$$\int_0^{34} r dt = \int_{683.4}^{1300} \frac{dQ(t)}{R(t)Q(t)} \tag{31}$$

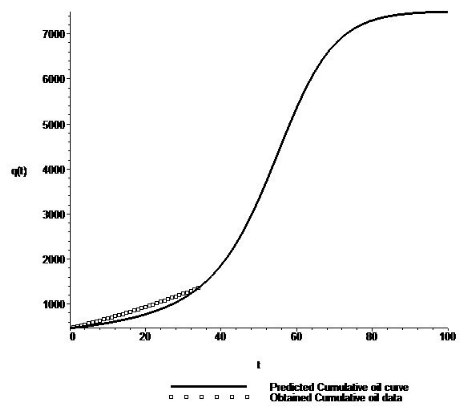

Figure 4: Our second model's curve of Quantity 0 in time is 1980

Which everything together results in an r of 2.9381×10^{-5}. Together with the a, b and c used above we use maple's DEtools and DEplot to result in the following plot for $Q(t)$.

Using the DEplot functions we produce the following graphs comparing $P(t)$, $Q(t)$ and $R(t)$, all of which where the line is the predicted, and the points are our data. Magenta is the colour for reserves $R(t)$, blue is for production $P(t)$ and black is for cumulative production $Q(t)$.

The second graph here is the data distributed over our period of thirty five years of data. Not noticeable at first, but is noticeable when looking at the graph below, production is not a good fit. This could be because of how we used to estimate production, or could be a failure of the validity of the model. Even then our values could have a better fit, but our constants were used to make a nice fit for the R versus Q graph and so might not have been best constants to decrease the error for when applied to time, but we needed the constants to be able to solve it.

From this graph we could even say that production looks linear for the current data given, so as long as the model appears linear in that range of values then the appropriate model is a good fit.

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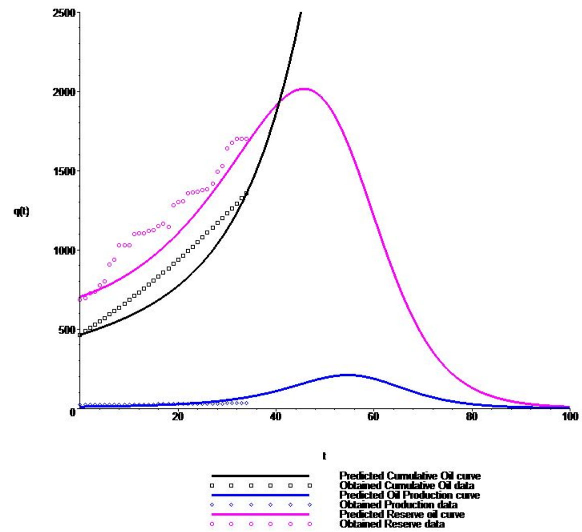

Figure 5: Our second model's graph, with production, reserves and cumulative quantity curves represented. 0 in time is 1980

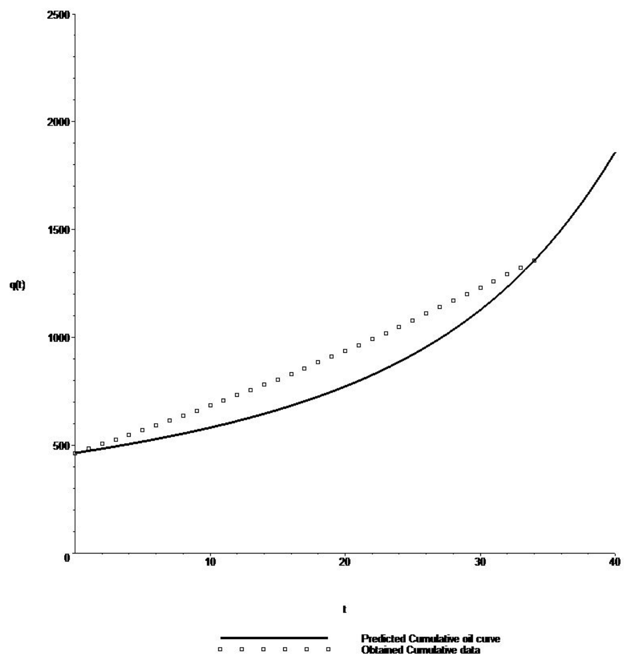

Figure 6: Our second model's graph, zoomed in on the first 35 years, so we can see the difference between data, and the curve.0 in time is 1980

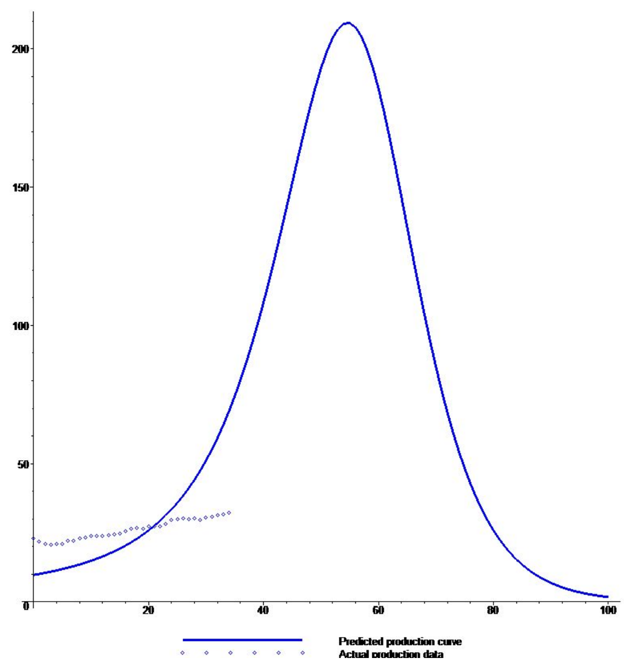

Figure 7: Our second model's graph, particularly looking at the difference in production. 0 in time is 1980

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